推薦入学(後期)試験問題
〈解答〉

[1] (1) $(2x^2+x+1)(x^2+x-1)$

x^3の係数 $2\times1+1\times1=2+1=3$

x^2の係数 $2\times(-1)+1\times1+1\times1=-2+1+1=0$

(2) $\dfrac{m\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\dfrac{n\sqrt{5}}{\sqrt{5}-\sqrt{3}}=2(\sqrt{15}-1)$

$$左辺=\frac{m\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}+\frac{n\sqrt{5}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$$

$$=\frac{m\sqrt{15}-3m}{2}+\frac{5n+n\sqrt{15}}{2}$$

$$=\frac{m+n}{2}\sqrt{15}+\frac{5n-3m}{2}$$

$$\begin{cases}\dfrac{m+n}{2}=2 & —① \\[2mm] \dfrac{5n-3m}{2}=-2 & —②\end{cases}$$

①より $m+n=4$ —①'

②より $5n-3m=-4$ —②'

①'×3+②' $8n=8$

$\underline{n=1}$

①に代入 $m+1=4$

$\underline{m=3}$

(3) $|x-5|=\frac{1}{2}x+1$

(i) $x\geqq5$のとき

$x-5=\frac{1}{2}x+1$

$\frac{1}{2}x=6$ $\therefore\ \underline{x=12}$ (答)

(ii) $x<5$のとき

$-(x-5)=\frac{1}{2}x+1$

$-\frac{3}{2}x=-4$

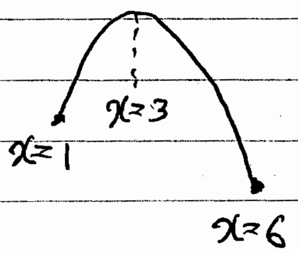
$\underline{x=\frac{8}{3}}$ (答)

(4) $y=-x^2+6x-6$

$=-(x^2-6x)-6$

$=-\{(x-3)^2-9\}-6$

$=-(x-3)^2+3$



$x=3$のとき $\underline{最大値\ 3}$

$x=6$のとき $\underline{最小値\ -6}$

(5) $0^\circ \leqq \theta \leqq 180^\circ$ のとき

$\tan\theta = -\frac{1}{2}$ だから θは鈍角 $(90^\circ < \theta < 180^\circ)$ $\therefore \sin\theta > 0,\ \cos\theta < 0$

$1 + \tan^2\theta = \frac{1}{\cos^2\theta}$ より

$$\cos^2\theta = \frac{1}{1+\tan^2\theta} = \frac{1}{1+\left(-\frac{1}{2}\right)^2} = \frac{1}{1+\frac{1}{4}} = \frac{4}{5}$$

$\cos\theta < 0$ より

$$\underline{\cos\theta = -\frac{2}{\sqrt{5}}}\ //$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ より

$$\sin\theta = \cos\theta\tan\theta = -\frac{2}{\sqrt{5}} \times \left(-\frac{1}{2}\right) = \underline{\frac{1}{\sqrt{5}}}\ //$$

[2] (1) $2a^2 - 2b^2 - 3c^2 - 3ab + 7bc - ac$

$= 2a^2 - (3b+c)a - (2b^2 - 7bc + 3c^2)$

$= 2a^2 - (3b+c)a - (2b-c)(b-3c)$

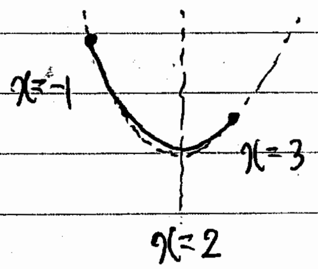
$$\begin{array}{ccc|c} 2 & \times & b-3c & b-3c \\ 1 & & -(2b-c) & -4b+2c \\ \hline & & & -3b-c \end{array}$$

$= \{2a + (b-3c)\}\{a - (2b-c)\}$

$= \underline{(2a+b-3c)(a-2b+c)}\ //$

(2) $x^2 - 4x - a^2 + 4a < 0$

$(x-2)^2 - 4 - a^2 + 4a < 0$



題意を満たすのは

$x=-1$ における最大値 < 0

$\therefore (-1)^2 - 4(-1) - a^2 + 4a < 0$

$-a^2 + 4a + 5 < 0$

$a^2 - 4a - 5 > 0$

$(a-5)(a+1) > 0$

$\underline{a < -1,\ 5 < a}\ //$

(3) $y=x^2+2ax+b$

$=(x+a)^2-a^2+b$

x軸の正の部分でx軸に接するから　$-a>0$　∴ $a<0$ —①

$-a^2+b=0$ —②

点(1,4)を通るから

$4=1+2a+b$ —③

②→③

$1+2a+a^2=4$

$a^2+2a-3=0$

$(a+3)(a-1)=0$

$a=-3,\ 1$

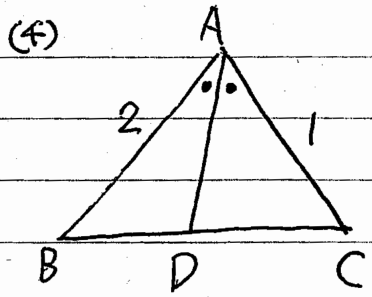
①より

$a=-3$

②に代入

∴ $b=(-3)^2=9$

(4)



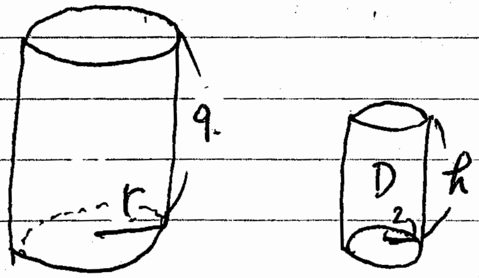
AB:AC=BD:DC=2:1だから

△ABDの面積は△ACDの面積の2倍

$BC=\frac{9}{4}$ ならば

$BD=\frac{\cancel{9}^{3}}{\cancel{4}_{2}}\times\frac{\cancel{2}}{\cancel{3}}=\frac{3}{2}$

(5)



$V_D=\pi\cdot 2^2h=4\pi h$

$V_E=\pi r^2\cdot 9=9\pi r^2$

$V_E=\frac{27}{8}V_D$ より

$\cancel{9}\cancel{\pi}r^2=\frac{\cancel{27}^{3}}{\cancel{8}_{2}}\cdot\cancel{4}\cancel{\pi}h$

$h=\frac{2}{3}r^2$ —①

$h:9=2:r$

$hr=18$ —②

①,②より　$\frac{\cancel{18}^{9}}{r}=\frac{\cancel{2}}{3}r^2$

$r^3=27$

∴ $r=3$

$h=\frac{2}{3}\times 3^2=6$

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[3] $C_1 : y = x^2 + 2x - 1$ $P(-2,-1) \cdot Q(1,2)$

$= (x+1)^2 - 2$

$C_2 : y = -ax^2 + bx + c \ (a>0)$

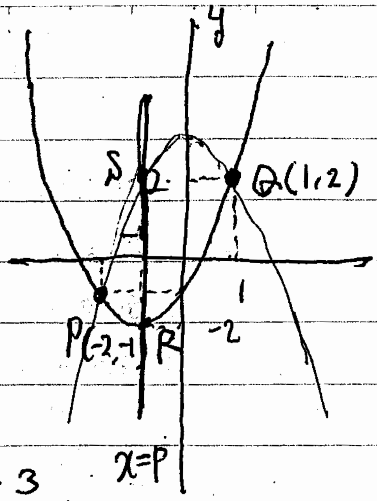
点 $P(-2,-1)$ を通るから $-1 = -4a - 2b + c$ —①

点 $Q(1,2)$ 〃 $2 = -a + b + c$ —②

①−② $-3 = -3a - 3b$

$\therefore a + b = 1 \quad \therefore \underline{b = 1 - a}$

②に代入 $2 = -a - 1 - a + c \quad \therefore \underline{c = 2a + 3}$ 〃



$R(p, p^2 + 2p - 1)$ $S(p, -ap^2 + bp + c)$

$RS = (-a-1)p^2 + (b-2)p + (c+1)$

$= (-a-1)p^2 + (1-a-2)p + (2a+3+1)$

$= -(a+1)p^2 - (a+1)p + 2(a+2)$

$= -(a+1)\{p^2 + p\} + 2(a+2)$

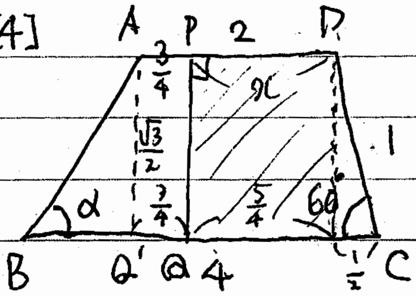
$= -(a+1)\{(p+\frac{1}{2})^2 - \frac{1}{4}\} + 2(a+2)$

$= -(a+1)(p+\frac{1}{2})^2 + \frac{1}{4}(a+1) + 2a + 4$

$= -(a+1)(p+\frac{1}{2})^2 + \frac{9}{4}a + \frac{17}{4}$

よって $\underline{p = -\frac{1}{2} \text{ のとき 最大値 } \frac{1}{4}(9a+17) \text{ をとる}}$ 〃

[4]



□ABCD の高さは $\sin 60° = \frac{\sqrt{3}}{2}$

$S = (2+4) \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \underline{\frac{3\sqrt{3}}{2}}$ 〃

□PQCD $= (x + x + \cos 60°) \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$= (2x + \frac{1}{2}) \cdot \frac{\sqrt{3}}{4} = \underline{\frac{(4x+1)\sqrt{3}}{8}}$ 〃

$\frac{(4x+1)\cancel{\sqrt{3}}}{\cancel{8}\,2} = \frac{3\cancel{\sqrt{3}}}{\cancel{4}}$

$4x + 1 = 6 \quad \underline{x = \frac{5}{4}}$ 〃

$\tan\alpha = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$ より

$BQ' = 4 - (\frac{3}{4} + \frac{5}{4} + \frac{1}{2}) = 4 - \frac{5}{2} = \frac{3}{2}$

$\underline{\alpha = 30°}$ 〃