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一般入学試験 数学I・II
<解答>

[1] (1) x^3-ax^2-5x-3 が x^2+bx-3 で割り切れる

$$\begin{cases}(a+b)b-2=0 & \text{―①}\\ a+b+1=0 & \text{―②}\end{cases}$$

$$\begin{array}{r} x+(-a-b) \\ x^2+bx-3 \overline{\big)\, x^3-ax^2-5x-3} \\ x^3+bx^2-3x \\ \hline (-a-b)x^2-2x-3 \\ (-a-b)x^2-(a+b)bx-3(-a-b) \\ \hline \{(a+b)b-2\}x-3(1+a+b) \end{array}$$

②より $a+b=-1$ ―③

①に代入して $-b-2=0$ $\therefore \underline{b=-2}$ 4

③に代入 $a-2=-1$ $\therefore \underline{a=1}$ 4

(2) $\dfrac{2-\sqrt{2}i}{1+\sqrt{2}i}+\dfrac{2}{1-\sqrt{2}i}=\dfrac{(2-\sqrt{2}i)(1-\sqrt{2}i)}{(1+\sqrt{2}i)(1-\sqrt{2}i)}+\dfrac{2(1+\sqrt{2}i)}{1+2}$

$=\dfrac{\cancel{2}-\cancel{2}-3\sqrt{2}i+2+2\sqrt{2}i}{3}=\underline{\dfrac{2-\sqrt{2}i}{3}}$ 4

$\therefore \underline{a=\dfrac{2}{3},\ b=-\dfrac{\sqrt{2}}{3}}$ 4

(3) $l: 4x+3y-6=0$ の傾きは $-\dfrac{4}{3}$ だから

点P$(4,2)$を通り lに直交する直線の傾きは $\dfrac{3}{4}$

よってその方程式は

$y-2=\dfrac{3}{4}(x-4)$

$\therefore 4y-8=3x-12$

$\underline{3x-4y-4=0}$

$\underline{y=\dfrac{3}{4}x-1}$ 4

P$(4,2)$と $l: 4x+3y-6=0$ との距離は

$d=\dfrac{|4\times4+3\times2-6|}{\sqrt{4^2+3^2}}=\underline{\dfrac{16}{5}}$ 4

(4) $0<\theta<2\pi$

$2\sin^2\theta+\cos\theta-1=0$

$2(1-\cos^2\theta)+\cos\theta-1=0$

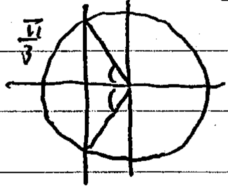
$-2\cos^2\theta+\cos\theta+1=0$

$2\cos^2\theta-\cos\theta-1=0$

$(2\cos\theta+1)(\cos\theta-1)=0$

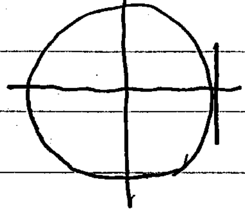
$\cos\theta=-\dfrac{1}{2},\ 1$

(i) $\cos\theta=-\dfrac{1}{2}$



$\theta=\dfrac{2}{3}\pi,\ \dfrac{4}{3}\pi$

(ii) $\cos\theta=1$



$\underline{なし}$ $(\theta\neq0,\ 2\pi)$

(5) $\dfrac{2^{-\frac{x}{2}}}{2^{\frac{x}{2}}-2^{-\frac{x}{2}}}-\dfrac{5\times 2^{\frac{x}{2}}}{2^{\frac{x}{2}}+2^{-\frac{x}{2}}}=-\dfrac{11}{3}$

左辺 $=\dfrac{2^{-\frac{x}{2}}(2^{\frac{x}{2}}+2^{-\frac{x}{2}})}{(2^{\frac{x}{2}})^2-(2^{-\frac{x}{2}})^2}-\dfrac{5\times 2^{\frac{x}{2}}(2^{\frac{x}{2}}-2^{-\frac{x}{2}})}{(2^{x}-2^{-x})}=\dfrac{1+\times 2^{-x}-5\times 2^{x}+5}{2^{x}-2^{-x}}$

$=\dfrac{1+2^{-x}-5\cdot 2^{x}+5}{2^{x}-2^{-x}}=-\dfrac{11}{3}$

$-11\cdot 2^{x}+11\cdot 2^{-x}=3\cdot 2^{-x}-15\cdot 2^{x}+18$

$4\cdot 2^{x}+8\cdot 2^{-x}=18$

$2^x=$ 大 とすると

$\cancel{4}^{2}大+\dfrac{\cancel{8}^{4}}{大}=\cancel{18}^{9}$

$2大^2-9大+4=0$

$(2大-1)(大-4)=0$

$\to$ 大 $=\dfrac{1}{2}, 4$

$\therefore 2^x=\dfrac{1}{2}, 4$

$\underline{x=-1, 2}$ //

[2] (1) $x^2+y^2-2x-4y+a=0$

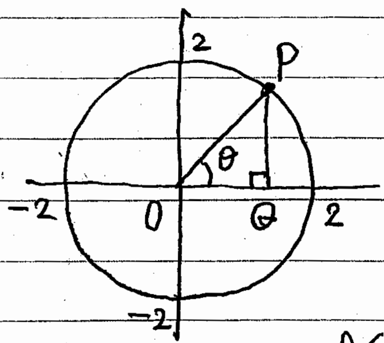
$(x-1)^2+(y-2)^2=5-a \quad (a<5)$

中心 $(1, 2)$ から $3x+4y+4=0$ との きょり d

$d=\dfrac{|3\times1+4\times2+4|}{\sqrt{3^2+4^2}}=\dfrac{15}{5}=\underline{3}$

$\therefore 5-a=9 \quad \underline{\therefore a=-4}$ //

(2)

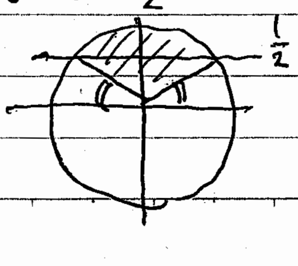


$0<\theta<\dfrac{\pi}{2}$

$P(2\cos\theta, 2\sin\theta)$

$\triangle POQ=2\cos\theta\times\cancel{2}\sin\theta\times\dfrac{1}{\cancel{2}}=\sin2\theta\geqq\dfrac{1}{2}$

$0<\theta<\dfrac{\pi}{2}$ より $0<2\theta<\pi$ だから



$\dfrac{\pi}{6}\leqq 2\theta\leqq\dfrac{5}{6}\pi$

$\underline{\dfrac{\pi}{12}\leqq\theta\leqq\dfrac{5}{12}\pi}$ //

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(3) $\log_3 x + 2\log_x 3 = 3$

真数条件より $x>0$ —①

$\log_3 x + \frac{2\log_3 3}{\log_3 x} = \log_3 x + \frac{2}{\log_3 x} = 3$

$(\log_3 x)^2 - 3(\log_3 x) + 2 = 0$

$(\log_3 x - 1)(\log_3 x - 2) = 0$

$\therefore \log_3 x = 1$ $\qquad \therefore \underline{x=3}$

$\log_3 x = 2$ $\qquad \therefore \underline{x = 3^2 = 9}$ //

(4) $f(x) = x^3 - 3x^2 + 3p^2x + p$

$f'(x) = 3x^2 - 6x + 3p^2$

極大値・極小値をもつから $D/4 = (-3)^2 - 9p^2 > 0$

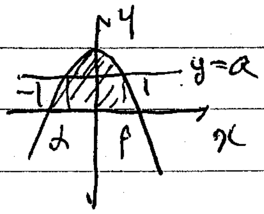
$9 - 9p^2 > 0$

$9(1+p)(1-p) > 0$

$(p+1)(p-1) < 0$

$\underline{-1 < p < 1}$ //

(5) $C: y = 1-x^2$



$S = \int_{-1}^{1}(1-x^2)dx = \left[x - \frac{1}{3}x^3\right]_{-1}^{1} = (1+1) - \frac{1}{3}(1+1) = 2 - \frac{2}{3} = \frac{4}{3}$

$\frac{1}{2} \times \frac{4}{3} = \int_{\alpha}^{\beta}(1-x^2-a)dx = \cancel{\left[(1-a)x - \frac{1}{3}x^3\right]_{\alpha}^{\beta}}$

$\frac{1}{6} = +\frac{1}{6}(\beta-\alpha)^3$

$1-\alpha^2 = a \Rightarrow \alpha^2 = 1-a$ α

$1-\beta^2 = a \Rightarrow \beta^2 = 1-a$

$\alpha<\beta$ より

$\alpha = -\sqrt{1-a},\ \beta = \sqrt{1-a}$

$\therefore (\beta-\alpha)^3 = (2\sqrt{1-a})^3 = 1$

$(1-a)^{\frac{3}{2}} = \frac{1}{8}$

$(1-a)^3 = \frac{1}{64} = \left(\frac{1}{4}\right)^3$

$1-a = \frac{1}{4}$ $\qquad \therefore \underline{a = \frac{3}{4}}$ //

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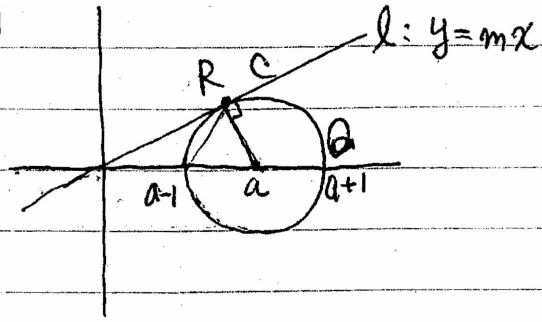
[3]



点(a,0)から $l: mx-y=0$ までの距離

$$d=\frac{|ma|}{\sqrt{m^2+1}}=1$$

$$ma=\sqrt{m^2+1}$$

$$m^2a^2=m^2+1$$

$$(a^2-1)m^2=1$$

$$m^2=\frac{1}{a^2-1}$$

$R(t, mt)$とおくと $(a,0)$との距離

$$d=\sqrt{(t-a)^2+(mt)^2}=1$$

$$t^2-2at+a^2+m^2t^2=1$$

$$t^2-2at+a^2+\frac{t^2}{a^2-1}=1$$

$$(a^2-1)t^2-2a(a^2-1)t+(a^2-1)^2+t^2=0$$

$$a^2t^2-2a(a^2-1)t+(a^2-1)^2=0$$

$$\begin{matrix} a & & a^2-1 \\ a & & a^2-1 \end{matrix}$$

$$(at-(a^2-1))^2=0$$

$$t=\frac{a^2-1}{a}=a-\frac{1}{a}$$

$$a+1=2\left(a-\frac{1}{a}\right)=2a-\frac{2}{a}$$

$$a^2+a-2a^2+2=0$$

$$a^2-a-2=0$$

$$(a-2)(a+1)=0$$

$a>1$より $a=2$

$a=2$のとき

$$m^2=\frac{1}{4-1}=\frac{1}{3}$$

$m>0$より $m=\frac{1}{\sqrt{3}}$

よって lとx軸のなす角は 30°

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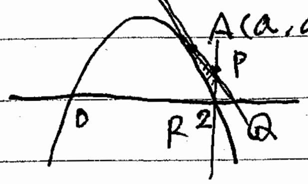
[4] $y = x(2-x) = -x^2+2x$



$y' = -2x+2$ より

Aにおける接線は

$y - a(2-a) = (-2a+2)(x-a)$

$y = (-2a+2)x + 2a^2 \cancel{-2a} \cancel{+2a} - a^2$

$y = (-2a+2)x + a^2 \cancel{+2a+2}$

$$S_1 = \int_a^2 \{(-2a+2)x + a^2 \cancel{+2a+2} + x^2 - 2x\}dx$$

$$= \int_a^2 (x^2 - 2ax + a^2 - 2a + 2)dx$$

$$= \left[\frac{1}{3}x^3 - ax^2 + (a^2-2a+2)x\right]_a^2$$

$$= \frac{1}{3}(8-a^3) - a(4-a^2) + (a^2-2a+2)(2-a)$$

$$= (2-a)\left\{\frac{1}{3}(2^2+2a+a^2) - a(2+a) + a^2 - 2a + 2\right\}$$

$$= (2-a)\left(\frac{4}{3} + \frac{2}{3}a + \frac{1}{3}a^2 - 2a \cancel{-a^2} \cancel{+a^2} - 2a + 2\right)$$

$$= (2-a)\left(\frac{1}{3}a^2 - \frac{10}{3}a + \frac{10}{3}\right)$$

$$= \frac{1}{3}(2-a)(a^2-10a+10)$$

$P(2,\ -4a+4+a^2 \cancel{+2a+2}) = \cancel{P(2,\ a^2-6a+6)}$

$Q\left(\dfrac{a^2-2a+2}{2(a-1)},\ 0\right)$ $R(2, 0)$

$$\triangle PQR = S_2 = \left(\frac{a^2-2a+2}{2(a-1)} - 2\right) \times (a^2-6a+6) \times \frac{1}{2}$$

$$= \frac{a^2-2a+2-4a+4}{2(a-1)} \times \frac{1}{2} \times (a^2-6a+6)$$

$$= \frac{(a^2-6a+6)^2}{4(a-1)}$$

$S_1 = S_2$ $\dfrac{(2-a)(a^2-10a+10)}{3} = \dfrac{(a^2-6a+6)^2}{4(a-1)}$

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$$\frac{(2-a)(a^2-10a+10)}{3} = \frac{(a^2-6a+6)^2}{4(a-1)}$$

$$\frac{-a^3+12a^2-30a+20}{3} = \frac{a^4+36a^2+36-12a^3+12a^2-72a}{4}$$

$$-4a^3+48a^2-120a+80 = 3a^4+108a^2+108-36a^3+36a^2-216a$$

$$3a^4-32a^3+\cancel{96}a^2-96a+28=0$$

$$\begin{array}{r} 144 \\ -48 \\ \hline 96 \end{array} \qquad \begin{array}{r} 216 \\ 120 \\ \hline 96 \end{array}$$

$$4(a^2-10a+10)(-a^2+3a-2) = 3(a^4+36a^2+36-12a^3-72a+12a^2)$$

$$-4(a^2-10a+10)(a^2-3a+2) = 3a^4+108a^2+108-36a^3-216a+36a^2$$

$$-4(a^4-13a^3+42a^2\underset{-50a}{}+20) = 3a^4-36a^3+144a^2-216a+108$$

$$-4a^4+52a^3-168a^2-80 = 3a^4-36a^3+144a^2-216a+108$$

$$7a^4-88a^3+312a^2\overset{+200a}{}-416a+188=0$$

$$S_1 = \int_a^2 \{(-2a+2)x+a^2+x^2-2x\}\,dx$$

$$= \int_a^2 (x^2-2ax+a^2)\,dx$$

$$= \left[\frac{1}{3}(x-a)^3\right]_a^2$$

$$= \frac{1}{3}\{(2-a)^3\} = \frac{(2-a)^3}{3}$$

$$S_2 = \left(\frac{a^2}{2(a-1)}-2\right)\times(a-2)^2\times\frac{1}{2}$$

$$= \frac{a^2-4a+4}{2(a-1)}\times(a-2)^2\times\frac{1}{2}$$

$$= \frac{(a-2)^4}{4(a-1)}$$

$P(2,(a-2)^2)$

$Q\left(\frac{a^2}{2(a-1)},0\right)$

$R(2,0)$

$S_1 = S_2$ より

$$\frac{(2-a)^3}{3} = \frac{(a-2)^4}{4(a-1)}$$

$$3(a-2)^4 = 4(2-a)^3(a-1)$$

$$(a-2)^2\{3(a-2)^2-4(2-a)(a-1)\}=0$$

$$(a-2)^2(3a^2-12a+12+4a^2-12a+8)=0$$

$$(a-2)^2(7a^2-24a+20)=0$$

$a \neq 2$ より

$$a = \frac{12\pm\sqrt{144-140}}{7} = \frac{12\pm2}{7} = 2,\ \frac{10}{7}$$

$1<a<2$ より

$$a=\frac{10}{7}$$